

Metric perturbations at reheating: the use of spherical symmetry

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Abstract

We consider decay of the inflaton with a quartic potential coupled to other fields, including gravity, but restricted to spherical symmetry. We describe analytically an early, quasilinear regime, during which inflaton fluctuations and the metric functions are driven by nonlinear effects of the decay products. We present a detailed study of the leading nonlinear effects in this regime. Results of the quasilinear approximation, in its domain of applicability, are found to be consistent with those of fully nonlinear lattice studies. We discuss how these results may be promoted to the full three dimensions.

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I. INTRODUCTION

Reheating is a crucial ingredient of inflationary cosmology [1]. In most models of inflation, the quasi-exponential expansion leaves the universe very cold and empty, with energy stored in coherent oscillations of the inflaton field. Some kind of nonlinear process is required to convert that energy into energy of ordinary particles.

Products of the inflaton decay (which will be denoted as χ but in general will include the short-wave fluctuations of the inflaton field itself) can cause metric perturbations in a variety of ways. Two possible processes are represented by diagrams of Fig. 1. In the first process, χ fluctuations scatter off the inflaton's homogeneous component (which is still large at that time), thereby producing inflaton fluctuations, which in turn cause perturbations of the metric. In the second process, χ fluctuations drive the metric perturbations directly.

We want to stress that the processes shown in the diagrams are quite generic and exist regardless of the particular mechanism of reheating. They exist even if the inflaton only decays perturbatively, although the scattering process of Fig. 1a is absent from the early-days perturbative treatment [2], in which the decay is represented by an effective friction term in the equation of motion for the inflaton field ϕ . Fig. 1a corresponds to a driving force in the equation of motion for ϕ that may allow fluctuations of ϕ to reach values $\delta\phi \sim \phi$. The question remains of course (which we will address in this paper) what portion of this large fluctuation resides in super-Hubble modes, in particular those relevant to structure formation.

Recent progress in the theory of reheating came after recognition that in many inflationary models the early stages of reheating can be studied rather completely. These are models where reheating starts by a parametric resonance [3,4]. Parametrically amplified fluctuations are almost classical, and nonlinear effects can be explored numerically in the classical approximation [5]. It was found that in many cases the resonant stage is followed by a stage (called semiclassical thermalization), during which nonlinear interactions of fluctuations become increasingly important, and power spectra of the fields eventually become smooth [5–7]. Importance of the scattering process of Fig. 1a was recognized early on in lattice studies in *rigid spacetimes* [5], and since then the estimate $\delta\phi \sim (0.1\text{--}1)\phi$ has been confirmed for a number of models [7–10].

Meanwhile, applications of the linear theory of metric perturbations to a special type of models have shown that in principle also super-Hubble modes, which necessarily activate *gravity*, can undergo growth at reheating [11–14]. It is therefore became important to combine the two lines of research and see if a similar super-Hubble growth can occur due to nonlinear effects. The first attempts in that direction were made by studies in restricted geometries: planar geometry in Ref. [15] and spherical symmetry in Ref. [16]. Both studies have indeed found such a growth [15,16].

The main purpose of the present paper is to supply an analytical counterpart to the numerical method of Ref. [16]. Although our approximate method, which we call *quasilinear* theory, cannot describe the fully nonlinear stage of the evolution, it allows us to describe the relevant nonlinear effects at earlier times, when the nonlinearity is still weak. This method retains linear terms in fluctuations of ϕ and the metric and quadratic terms in χ . It generalizes the method developed in Refs. [7,10] for rigid spacetimes to the case when metric perturbations are present. In particular, it will allow us to take into account the

processes shown in Fig. 1, which are not included in the standard linear theory of metric perturbations. (In this paper, we consider the case when the expectation value of χ is initially zero, so these nonlinear effects are the leading effects of χ .)

The system of spherically-symmetric equations is written down in Sect. II. In Sect. III, we construct the quasilinear theory, restricting our attention to a model with a quartic inflaton potential. Within the quasilinear theory, we identify two potential sources of metric perturbations at super-Hubble scales. The first is the “erosion” of the inflaton by χ fluctuations, as shown in Fig. 1a; the second, is the metric perturbations caused by the anisotropic stress, a purely relativistic effect that can be viewed as a part of Fig. 1b. In the subsequent sections we analyze these effects in turn. We find that unless the decay products themselves have infrared-singular spectra the “erosion” is not infrared singular and thus cannot be important on scales relevant to structure formation. On the other hand, the anisotropic stress gives rise to long-range (secular) terms in the gravitational potentials and is important at large scales, in a calculation restricted to spherical symmetry. In the concluding Sect. VIII, we discuss the overall physical picture that emerges when we try to promote our findings to the full three dimensions.

II. GRAVITATING SCALAR FIELDS IN SPHERICAL SYMMETRY

Early on in reheating, the particle density is small, and a weakly interacting particle species has a very large mean-free path. For such a species, a spherically-symmetric field will be relatively stable with respect to fragmentation into higher partial waves. This means that the restriction to spherical symmetry, when it is desired or unavoidable, can be imposed directly on the fields (and the metric), rather than on compound quantities, such as the stress tensor. In this sense, our calculations will be quite different from those with spherically-symmetric *fluids*: in the latter case particles move in all directions; it is only the large-scale motion of the fluid that is radial. In our case, because of the large mean-free paths, the fluid approximation is not readily available.

We consider a system of scalar fields minimally coupled to gravity and described by the Lagrangian

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \sum_i g_{\mu\nu} \partial^\mu \phi_i \partial^\nu \phi_i - V \right] \quad (1)$$

where i labels different fields, $\kappa^2 = 8\pi G$, and $V = V(\phi_i)$ is the potential which can depend on all of them. The most general spherically symmetric metric is

$$ds^2 = (-N^2 + \psi^4 \beta^2) dt^2 + 2\psi^4 \beta dt dr + \psi^4 dr^2 + \alpha^2 r^2 d\Omega^2 \quad (2)$$

where $d\Omega^2$ is the line element of the 2-sphere and all the four functions N, ψ, α, β depend on t and r . We shall work in *isotropic* coordinates [17] ($\alpha^2 = \psi^4$). Accordingly we shall have only one conjugate variable to ψ by a diagonal gauge in which the shift vector β is set to zero: in this way the extrinsic curvature will be determined only by its trace [17]. Therefore the metric we shall use is the following

$$ds^2 = -N^2(t, r) dt^2 + \psi^4(t, r) [dr^2 + r^2 d\Omega^2] . \quad (3)$$

The equation of motion for a scalar field with a generic potential $V(\phi)$

$$-\nabla^\mu \partial_\mu \phi_i + \frac{\partial V}{\partial \phi_i} = 0 \quad (4)$$

are written in a Hamiltonian form for the metric (3)

$$\dot{\phi}_i = N \frac{\pi_i}{\psi^6} \quad (5)$$

$$\dot{\pi}_i = \frac{1}{r^2} \left(r^2 \psi^2 N \phi_i' \right)' - N \psi^6 \frac{\partial V}{\partial \phi_i} \quad (6)$$

where ϕ, π_i are the pair of conjugate variables for the scalar field, a dot denotes the derivative with respect to t whereas a prime denotes the derivative with respect to r .

The Hamiltonian equations for the metric are the following:

$$\dot{\psi} = -\frac{N\psi}{6} K \quad (7)$$

$$\frac{\dot{K}}{N} = \frac{K^2}{2} - 6 \frac{\psi'}{\psi^5} \left(\frac{\psi'}{\psi} + \frac{1}{r} \right) - 3 \frac{N'}{N\psi^4} \left(\frac{1}{r} + 2 \frac{\psi'}{\psi} \right) + \frac{3}{2} \kappa^2 E - 3\kappa^2 V \quad (8)$$

where K is the trace of the extrinsic curvature and E is the total Hamiltonian for the scalar fields

$$E = \sum_i \left(\frac{\pi_i^2}{2\psi^{12}} + \frac{\phi_i'^2}{2\psi^4} \right) + V. \quad (9)$$

The equation for the lapse function N does not involve time derivatives (since N is not a dynamical degree of freedom):

$$\frac{r\psi^4}{N} \left(\frac{N'}{r\psi^4} \right)' - 6 \frac{\psi'^2}{\psi^2} + \frac{2r}{\psi} \left(\frac{\psi'}{r} \right)' = -\kappa^2 \sum_i \phi_i'^2 \quad (10)$$

The energy and momentum constraints are

$$\frac{1}{2} {}^{(3)}R = -\frac{4}{\psi^5} \nabla^2 \psi = \left[\kappa^2 E - \frac{K^2}{3} \right] \quad (11)$$

$$\frac{2}{3} K' = \sum_i \kappa^2 \frac{\pi_i \phi_i'}{\psi^6} \quad (12)$$

where ${}^{(3)}R$ is the curvature of spatial sections.

III. QUASILINEAR REGIME

To describe analytically the phase that follows the linear stage, we employ now a *quasi-linear* approximation, which generalizes the method developed in [7,10] for rigid spacetimes. In this approximation we consider terms that are linear in the fluctuations of the inflaton and of the metric, but keep quadratic terms in χ fluctuations. In the absence of an initial

homogeneous value of χ , these quadratic terms represent the leading effects of χ on the inflaton and the metric functions.

The metric at the first order has the same general form as in the linear theory (reviewed in Ref. [18]). Including only scalar perturbations but keeping (for now) three-dimensional coordinates, we write it in the longitudinal gauge as

$$ds^2 = a^2(t) \left\{ -[1 - 2\Phi(t, \mathbf{x})]dt^2 + [1 + 2\Psi(t, \mathbf{x})]d\mathbf{x}^2 \right\} ; \quad (13)$$

t now is the conformal time. An important difference with the linear theory, however, is that even for minimally coupled scalars, there is an *anisotropic stress*, induced by a nonlinear contribution of χ . As a result, $\Phi \neq \Psi$.

For definiteness, from now on we consider a model with two fields: the inflaton ϕ and the second field χ whose fluctuations will drive fluctuations of ϕ and the metric. Both fields are minimally coupled to gravity, and their potential is

$$V(\phi, \chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{4}\lambda'\chi^4 . \quad (14)$$

Our method applies equally well to single field models, in which case the role of χ will be played by short-scale components of the inflaton itself.

To the linear order in metric perturbations, equations for the fields ϕ and χ can be obtained without restriction to spherical symmetry. On the other hand, equations for the metric functions we present below are valid only under that restriction.

Expanding the equation of motion for ϕ to the first order in Φ , Ψ , χ^2 , and the spatial derivatives of ϕ we obtain

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + \nabla^2\phi + \lambda a^2\phi^3 + g^2 a^2\phi\chi^2 = -\dot{\phi}(\dot{\Phi} + 3\dot{\Psi}) + 2\lambda a^2\phi^3\Phi , \quad (15)$$

where $\mathcal{H} = \dot{a}/a$; dots denote derivatives with respect to t . We can separate the field ϕ into its volume average, ϕ_0 , and a fluctuation:

$$\phi(t, \mathbf{x}) = \phi_0(t) + \delta\phi(t, \mathbf{x}) . \quad (16)$$

The volume averages of Φ and Ψ are made zero through definitions of the scale factor a and the conformal time. We also assume that the volume average of the field χ is initially zero.

Substituting (16) into (15), expanding to the first order, and averaging over the volume gives

$$\ddot{\phi}_0 + 2\mathcal{H}\dot{\phi}_0 + \lambda a^2\phi_0^3 + g^2 a^2\phi_0\overline{\chi^2} = 0 ; \quad (17)$$

the overline denotes a volume average. Subtracting (17) from (15), we obtain an equation for the fluctuation:

$$\delta\ddot{\phi} + 2\mathcal{H}\delta\dot{\phi} + \left[-\nabla^2 + 3\lambda a^2\phi_0^2 \right] \delta\phi + g^2 a^2\phi_0(\chi^2 - \overline{\chi^2}) = -\dot{\phi}_0(\dot{\Phi} + 3\dot{\Psi}) + 2\lambda a^2\phi_0^3\Phi . \quad (18)$$

Similarly, for χ , to the first order in Φ , Ψ , χ^2 , and $\delta\phi$, we obtain

$$\ddot{\chi} + 2\mathcal{H}\dot{\chi} + \left[-\nabla^2 + g^2 a^2\phi_0^2 \right] \chi = -2g^2 a^2\phi_0\chi\delta\phi - \dot{\chi}(\dot{\Phi} + 3\dot{\Psi}) . \quad (19)$$

In this paper, we use quasilinear theory only for the earliest times, when the right-hand of (19) can be neglected. In this case, we obtain an equation for χ which is unaffected by metric perturbations:

$$\ddot{\chi} + 2\mathcal{H}\dot{\chi} + \left[-\nabla^2 + g^2 a^2 \phi_0^2\right] \chi = 0. \quad (20)$$

In the full 3d case, the nonlinearities of χ would act as a source not only for scalar metric perturbations, but also for vector and tensor ones. Equations given below are restricted to spherical symmetry, in which case only scalar perturbations are present.

In the quasilinear approximation, the energy constraint (12) reads

$$-\nabla^2 \Psi + 3\mathcal{H}(\dot{\Psi} + \mathcal{H}\Phi) = \frac{\kappa^2}{2} (\delta E_\phi + \delta E_\chi) \quad (21)$$

where the fluctuations of energy densities of the fields are defined by $\delta E_i = E_i - \overline{E_i}$; for the model at hand, to the required order,

$$\delta E_\phi = \dot{\phi}_0 \delta \dot{\phi} + \dot{\phi}_0^2 \Phi + a^2 \lambda \phi_0^3 \delta \phi, \quad (22)$$

$$E_\chi = \frac{\dot{\chi}^2}{2} + \frac{(\nabla \chi)^2}{2} + \frac{g^2}{2} a^2 \phi_0^2 \chi^2. \quad (23)$$

The momentum constraint (12) reads

$$(\dot{\Psi} + \mathcal{H}\Phi)' = -\frac{\kappa^2}{2} (\dot{\phi}_0 \delta \phi' + \dot{\chi} \chi'). \quad (24)$$

We do not present a quasilinear form for Eq. (8). The quasilinear form for Eq. (10) is:

$$r \left(\frac{(\Phi - \Psi)'}{r} \right)' = \kappa^2 \chi'^2. \quad (25)$$

The right-hand side is the anisotropic stress.

In general, an *anisotropic stress* is generated by a non-vanishing traceless part of the spatial energy-momentum tensor. In our case, it is due to the traceless second-order contribution of the field χ :

$$\Pi_j^i = \partial^i \chi \partial_j \chi - \frac{1}{3} \delta_j^i \partial^l \chi \partial_l \chi. \quad (26)$$

For small scalar perturbations, the corresponding Einstein equation takes

$$(D^i D_j - \frac{1}{3} \delta_j^i D^2) [\Phi - \Psi] = \kappa^2 [\Pi_j^i]_S, \quad (27)$$

where D_i is the covariant derivative with respect to the spatial metric γ_{ij} , related to g_{ij} via $g_{ij} = \psi^4 \gamma_{ij}$, $D^2 = D^i D_i$, and the subscript S denotes the scalar part of Π_j^i . The scalar part is automatically selected when we make restriction to spherical symmetry, and the result coincides with Eq. (25).

We see that nonlinearities in χ can potentially cause two types of effects. First, the rescattering term—the last term on the right-hand side of (18)—will lead to a growth of $\delta \phi$; this is the effect represented by the diagram of Fig. 1. Second, the right-hand side of Eq. (25) directly produces $\Phi - \Psi \neq 0$. Either way, the resulting perturbations are non-Gaussian. In the remaining sections, we will study these two effects in turn. Note that *neither* of these effects was taken into account in Ref. [19] where only the direct second-order contribution of χ to curvature perturbation was considered (in a different model).

IV. INFLATON FLUCTUATIONS

The two effects described at the end of the previous section are more easily disentangled if we use, instead of $\delta\phi$, the gauge-invariant variable

$$Q_\phi = \delta\phi - \dot{\phi}\Psi/\mathcal{H}. \quad (28)$$

This variable was introduced by Mukhanov to characterize perturbations produced *during* inflation [20]; it was applied to studies of reheating at the linear level in Refs. [21,12–14].¹ From now on we omit the zero subscript on the homogeneous component of ϕ , hoping that this will not cause any confusion.

Using Eq. (18) and the quasilinear equations for the metric, we obtain the following equation for Q_ϕ :

$$\ddot{Q}_\phi + 2\mathcal{H}\dot{Q}_\phi + \left[-\nabla^2 + 3\lambda a^2\phi^2 + \omega_{\text{GRAV}}^2\right] Q_\phi + g^2 a^2 \phi \delta\chi^2 = F(\chi), \quad (29)$$

where $\delta\chi^2 = \chi^2 - \overline{\chi^2}$, the gravitational contribution ω_{GRAV}^2 to the frequency is

$$\omega_{\text{GRAV}}^2 = \kappa^2 \dot{\phi} \left(2a^2 \frac{V_{,\phi}}{\mathcal{H}} + 3\dot{\phi} - \kappa^2 \frac{\dot{\phi}^3}{2\mathcal{H}^2} \right), \quad (30)$$

and V is the potential of ϕ ; in our case, $V_{,\phi} = \lambda\phi^3$. This potential does not include a Hartree correction, cf. Eq. (17). Indeed, including such a correction here would take us outside of the quasilinear approximation. Thus, (30) is the standard expression of the linear theory [20,18,12]. The force term $F(\chi)$ depends only on χ :

$$F(\chi) = \kappa^2 \frac{\dot{\phi}}{\mathcal{H}} \left[-\frac{1}{3}(\nabla\chi)^2 - \frac{1}{2}g^2 a^2 \phi^2 \delta\chi^2 + \frac{1}{3\kappa^2} \nabla^2(\Phi - \Psi) \right] - \left(6\dot{\phi} + 2a^2 \frac{V_{,\phi}}{\mathcal{H}} - \kappa^2 \frac{\dot{\phi}^3}{\mathcal{H}^2} \right) \mathcal{Z}_\chi, \quad (31)$$

where \mathcal{Z}_χ is the contribution of χ to the curvature perturbation, see eqs. (46), (47) below. The term proportional to $\nabla^2(\Phi - \Psi)$ depends only on χ by virtue of the quasilinear approximation, see Eq. (25).

Because Eq. (29), unlike Eq. (18), depends on the gravitational potentials only through $\nabla^2(\Phi - \Psi)$, the most long-range (secular) contributions to Φ and Ψ do not enter, and all interactions generated by gravity in Eq. (29) are suppressed by at least one power of M_{Pl} . (This suppression is in parallel with the result of Ref. [13] for the linear case.) Neglecting all such terms and going over to the variable $X = aQ_\phi$, we obtain

$$\ddot{X} + (-\nabla^2 + 3\lambda a^2\phi^2)X = -g^2 a^3 \phi (\chi^2 - \overline{\chi^2}), \quad (32)$$

¹This variable looks as a combination of metric and field fluctuations in the longitudinal gauge, while in the uniform curvature gauge its interpretation is more transparent: it is simply the field fluctuation, which indeed is the true dynamical variable of the problem (see for example [22]).

where we have neglected the term \ddot{a}/a in the frequency because a radiation dominated universe ($\ddot{a} \approx 0$) is a good approximation during reheating in this model. Eq. (32) shows that, in the quasilinear theory, the growth of X is driven primarily by rescattering due to the local coupling and is essentially unaffected by gravity.

By using the spherically symmetric equations for gravity in deriving Eq. (32), we have neglected vector and tensor perturbations. It seems plausible, however, that contributions of these types of perturbations to the evolution of Q_ϕ are suppressed in the same way as the contribution of scalar perturbations. We therefore expect that Eq. (32) applies, to the leading order in M_{Pl}^{-1} , in the full three dimensions. Accordingly, we write it in the Fourier space as

$$\ddot{X}_{\mathbf{k}} + (k^2 + 3\lambda a^2 \phi^2)X_{\mathbf{k}} = -g^2 a^3 \phi \int d^3 p \chi_{\mathbf{p}}(t') \chi_{\mathbf{k}+\mathbf{p}}(t'). \quad (33)$$

The solution to Eq. (33) can be written in terms of the Green function:

$$X_{\mathbf{k}}(t) = X_{\mathbf{k},\text{INFL}}(t) - g^2 \int_0^t dt' G_{\mathbf{k}}(t, t') a^3(t') \phi(t') \int d^3 p \chi_{\mathbf{p}}^*(t') \chi_{\mathbf{k}+\mathbf{p}}(t'), \quad (34)$$

where $X_{\mathbf{k},\text{INFL}}(t)$ is the solution to the linear problem, corresponding to perturbations produced during inflation and the crossover to reheating. The second, forced, term in Eq. (34) starts from zero at $t = 0$ but then grows, initially as $\exp(2\mu_{\text{max}} t)$ where μ_{max} is the maximal Floquet exponent for χ . There is no additional decay in time, since the factor a^3 cancels the damping envelope $\sim 1/a$ of the fields ϕ, χ .

The Green function can be written as

$$G_{\mathbf{k}}(t, t') = \frac{X_{k1}^{(0)}(t') X_{k2}^{(0)}(t) - X_{k1}^{(0)}(t) X_{k2}^{(0)}(t')}{W_k}, \quad (35)$$

where $X_{k1,2}^{(0)}$ are two independent solutions to the homogeneous equation

$$\ddot{X}_{k1,2}^{(0)}(t) + (k^2 + 3\lambda a^2 \phi^2) X_{k1,2}^{(0)}(t) = 0, \quad (36)$$

and W_k is their (time independent) Wronskian. The homogeneous solutions are known in implicit form [23] (see also [24]):

$$X_{k1,2}^{(0)}(t) = \sqrt{|M_k(t)|} \exp\left(\pm \frac{W_k}{2} \int \frac{dt}{M_k(t)}\right), \quad (37)$$

where $M_k(t)$ is defined in Eq. (37) of Ref. [23], and the Wronskian W_k is

$$W_k = \frac{8}{9} k \sqrt{(x^4 - \frac{9}{4})(3 - x^4)} \quad (38)$$

with $x = k/(\sqrt{\lambda}\phi(0))$.

The Wronskian W_k is real in the single resonance band $3/2 < x^2 < \sqrt{3}$ and imaginary everywhere else. When W_k is imaginary, $W_k = i|W_k|$, the solutions (37) are complex conjugate, and the Green function becomes

$$G_k(t, t') = \frac{2}{|W_k|} \sqrt{|M_k(t)M_k(t')|} \sin \left(\frac{|W_k|}{2} \int_t^{t'} \frac{dt''}{M_k(t'')} \right). \quad (39)$$

At small k , $|W_k| \sim k$, while $M_k(t)$ is finite and nonzero, so G_k is regular there. Thus, if the field χ has not been amplified during inflation, the integral over \mathbf{p} in Eq. (34) remains regular at $k = 0$ as long as the quasilinear approximation applies.

Using Eq. (34), we can estimate the power spectrum of the forced (non-Gaussian) part of Q_ϕ as

$$P_{Q,\text{NG}}(k) \sim \left(\frac{g^2}{\lambda\phi} \right)^2 \int d^3p P_\chi(p) P_\chi(|\mathbf{p} + \mathbf{k}|), \quad (40)$$

where P_χ is the power spectrum of χ . We should now distinguish two cases.

(i) If the field χ was light during inflation, $g^2 \lesssim \lambda$, the spectrum of χ produced at that time is infrared singular, which by Eq. (40) gives rise to an infrared-singular spectrum of Q_ϕ . This will result in non-Gaussian adiabatic perturbations of the metric at scales of relevance to structure formation, in addition to the previously considered [25] perturbations due to a direct quadratic contribution of χ to the energy density.

For $1 < g^2/\lambda < 3$, when the infrared part of the spectrum is amplified by parametric resonance, this effect can be quite large. It is the largest for $g^2/\lambda = 2$, when the resonance peak is centered at $k = 0$. In this case, assuming for simplicity the exact scale-invariant spectra for both ϕ and χ at the end of inflation, we can estimate P_χ as $P_\chi(k) \sim \langle \chi^2 \rangle_{\text{full}}/k^3$, where $\langle \chi^2 \rangle_{\text{full}}$ denotes the full 3d variance. The resulting $P_{Q,\text{NG}}$ should be compared to the inflation-produced Gaussian spectrum $P_{Q,\text{INFL}} \sim H_{\text{INFL}}^2/k^3$, where H_{INFL} is the Hubble parameter during inflation. Using the maximal value of the variance $\langle \chi^2 \rangle_{\text{full}} \sim \lambda\phi^2/g^2$ [7,8], we obtain

$$\frac{P_{Q,\text{NG}}}{P_{Q,\text{INFL}}} \sim \left(\frac{g^2}{\lambda\phi} \right)^2 \frac{\langle \chi^2 \rangle_{\text{full}}^2}{H_{\text{INFL}}^2} \sim \frac{\phi^2}{H_{\text{INFL}}^2} \sim 10^6. \quad (41)$$

(A similar, but Gaussian, effect is obtained in linear theory in certain models with a nonvanishing homogeneous component of χ [13,14].)

(ii) If the field χ was heavy during inflation, it arrives at reheating with a constant (white-noise) spectrum at small k , and by Eq. (40) the spectrum of Q_ϕ is also constant (“non-Gaussian white noise”). Indeed, assuming that the resonance peak is located at $k = k_*$, we estimate that $P_\chi \sim \langle \chi^2 \rangle_{\text{full}}/k_*^3$, where $\langle \chi^2 \rangle_{\text{full}}$ denotes the full 3d variance. Then,

$$P_{Q,\text{NG}} \sim \left(\frac{g^2}{\lambda\phi} \right)^2 \frac{\langle \chi^2 \rangle_{\text{full}}^2}{k_*^3}. \quad (42)$$

Using again the maximal value $\langle \chi^2 \rangle_{\text{full}} \sim \lambda\phi^2/g^2$, we obtain

$$P_{Q,\text{NG}} \sim \phi^2/k_*^3. \quad (43)$$

This estimate can be converted into an estimate for curvature perturbations, as measured by the parameter ζ ,

$$(1+w)\zeta = \frac{2}{3} \left(\dot{\Psi}/\mathcal{H} + \Phi \right) + (1+w)\Psi , \quad (44)$$

where

$$1+w = \frac{\kappa^2 \dot{\phi}^2}{3\mathcal{H}^2} . \quad (45)$$

The term $1+w$ in front of ζ is needed in order to avoid singularity at $\dot{\phi} = 0$ [12,13]. Since $1+w$ just oscillates between finite values, a growth of the left-hand side of (44) can only come from a growth of ζ .

The momentum constraint (24), can be generalized to the full three dimensions; the resulting scalar part, separated along the lines of Ref. [26], is

$$\dot{\Psi}_{\mathbf{k}} + \mathcal{H}\Phi_{\mathbf{k}} = -\frac{\kappa^2}{2} \left(\dot{\phi}\delta\phi_{\mathbf{k}} + \frac{1}{k^2} \int d^3p (\mathbf{k} \cdot \mathbf{p}) \dot{\chi}_{\mathbf{p}+\mathbf{k}} \chi_{\mathbf{p}}^* \right) \equiv -\frac{\kappa^2}{2} \dot{\phi}\delta\phi_{\mathbf{k}} - \mathcal{Z}_{\chi\mathbf{k}} . \quad (46)$$

Using (46), we can express ζ as

$$(1+w)\zeta_{\mathbf{k}} = -\frac{\kappa^2}{3\mathcal{H}} \dot{\phi} Q_{\phi\mathbf{k}} - \frac{2}{3\mathcal{H}} \mathcal{Z}_{\chi\mathbf{k}} . \quad (47)$$

At small k , we estimate the integral in (46), in case (ii), as

$$\int d^3p (\mathbf{k} \cdot \mathbf{p}) \dot{\chi}_{\mathbf{p}+\mathbf{k}} \chi_{\mathbf{p}}^* \sim \mathcal{O}(k^2) , \quad (48)$$

so that the direct contribution of χ in this case is independent of k in the small- k limit. The same conclusion was reached previously in Ref. [19], albeit for a different model and by a different method. However, comparing (47) and (40), we see that at $g^2 \gg \lambda$ the contribution from Q_{ϕ} , which was not considered in Ref. [19], is parametrically larger than the direct contribution of χ .

Even that larger contribution is still quite small. Using (43), we obtain

$$P_{\zeta,\text{NG}}(k) \sim \frac{\kappa^4 \dot{\phi}^2 \phi^2}{\mathcal{H}^2 k_*^3} \sim \frac{\phi^2}{M_{\text{Pl}}^2 k_*^3} , \quad (49)$$

so that the perturbation of ζ on scales larger than the Hubble scale is

$$\delta_H \zeta \sim \frac{\phi}{M_{\text{Pl}}} \left(\frac{\mathcal{H}}{k_*} \right)^{3/2} \sim 10^{-6} , \quad (50)$$

where we have used $\phi/M_{\text{Pl}} \sim 3 \times 10^{-3}$ and $\mathcal{H}/k_* \sim 6 \times 10^{-3}$. We doubt that such a small perturbation can cause a gravitational instability, such as one observed numerically in Ref. [16]. Indeed, in the following sections we will identify a much stronger infrared effect, due to anisotropic stress.

The form (39) of the Green function leaves open the possibility of $\delta\phi$ cascading towards the infrared. Indeed, if the integral in the argument of sine increases with time, the region of k where the regular small- k limit applies will be pushed further and further in the infrared. However, the corresponding growth of the spatial scale of the perturbations can only occur at the relevant excitation speed, such as the speed of sound, and so cannot overtake the cosmological horizon (which recedes at the speed of light).

V. INITIALIZATION OF A RADIAL FIELD

It is clear from Eq. (25) that to understand the effect of the anisotropic stress, we need to understand the spatial profile of the field χ . It is useful to look first at the initial state, because the profile of χ remains qualitatively the same throughout the quasilinear stage of the evolution.

A spherically-symmetric (also called radial) field χ is initialized according to the formula²

$$r\chi(r) = A \sum_k s_k k P_\chi^{1/2}(k) \sin kr , \quad (51)$$

where k are the lattice momenta, quantized in units of π/R , s_k are Gaussian random numbers with zero average and unit dispersion, and P_χ is the power spectrum. The constant coefficient A is to be chosen upon deciding what part of the full three-dimensional (3d) variance the radial field will account for. The latter question arises because in the full 3d case there surely will be other, nonradial, modes, which will carry a part (in fact, most) of the variance.

Consider the average of the field defined in (51) over the distribution of the random numbers s_k :

$$\langle \chi^2 \rangle(r) = \frac{A^2}{r^2} \frac{R}{\pi} \int_{k_{\min}}^{k_{\max}} dk k^2 P_\chi(k) \sin^2 kr , \quad (52)$$

where we have replaced the sum with an integral, and k_{\min} and k_{\max} are the minimal and maximal lattice momenta. For a field that was heavy during inflation, we use $P_\chi(k)$ corresponding to the adiabatic vacuum, with an additional ultraviolet cutoff:

$$P_\chi(k) = (2\pi)^{-3} (2\omega_k)^{-1} \exp(-k^2/k_0^2) . \quad (53)$$

where ω_k is the instantaneous frequency at $t = 0$. For this power spectrum, the average (52) has the asymptotic form

$$\langle \chi^2 \rangle(r) \approx \frac{A^2}{r^2} \frac{R}{2\pi} \int P_\chi(k) k^2 dk \quad (54)$$

at large r . On the other hand, the full 3d variance is independent of the coordinates and equals

$$\langle \chi^2 \rangle_{\text{full}} = 4\pi \int P_\chi(k) k^2 dk . \quad (55)$$

The field χ is smooth, almost constant, over a patch of radius $r \ll \pi/k_0$, where k_0 is the ultraviolet cutoff from (53). A smooth field is predominantly an s -wave, so for such r we expect the average (52) to approximately equal the full 3d variance (55). In a sense, the scale

² In numerical work, after the field χ has been initialized according to (51) we additionally subtract away its homogeneous component. This does not matter for the present argument.

$$\ell_0 = \pi/k_0 \quad (56)$$

can be viewed as the initial correlation length of the field. The asymptotic expression (54) applies at $r \gg \ell_0$, but as r approaches ℓ_0 , we expect $\langle \chi^2 \rangle(r)$ to approach $\langle \chi^2 \rangle_{\text{full}}$. Accordingly, we adopt the following estimate for the magnitude of $\langle \chi^2 \rangle(r)$ at $r \gg \ell_0$:

$$\langle \chi^2 \rangle(r) = \langle \chi^2 \rangle_{\text{full}} \ell_0^2 / r^2. \quad (57)$$

This corresponds to

$$A^2 = 8\pi^2 \ell_0^2 / R. \quad (58)$$

A number of order one could be inserted here, but it has practically no effect on the final results. The important point is that, at a fixed r , Eq. (57) gives the size of fluctuations that is independent of the spatial volume. Equivalently, the volume average

$$\frac{4\pi}{V} \int_0^R r^2 dr \langle \chi^2 \rangle(r) = \frac{4\pi \ell_0^2 R}{V} \langle \chi^2 \rangle_{\text{full}}, \quad (59)$$

decreases with R as $1/R^2$.

The $1/r^2$ dependence of the variance of a radial field can be simply interpreted as follows. For an observer at the origin, a smooth patch of size ℓ_0 at distance $r \gg \ell_0$ has angular size ℓ_0/r . Thus, expansion of a 3d field with correlation length ℓ_0 will require of order $(r/\ell_0)^2$ different spherical harmonics, with amplitudes that are roughly of the same order. The variance in each of these partial waves, specifically the s -wave, will then account for $(\ell_0/r)^2$ of the full 3d variance. Similar considerations apply to perturbations of the metric.

In Ref. [16], in preparation of the initial state, we in effect assumed that the volume averages of $\langle \chi^2 \rangle$ and of the variances of the metric functions, e.g. the left-hand side of (59), equaled the full 3d variances. As we now see, that was an overestimate of the size of initial fluctuations. However, as typical in problems where amplification of fluctuations is cut off by nonlinear effects, the initial size is relatively unimportant: repeating the run of Ref. [16] with initial data of the correct size, we find that the strong gravitational effects, which we interpreted as formation of a black hole, are merely postponed from $t \approx 74$ to $t \sim 105$ (in the units of Ref. [16]).

VI. EFFECT OF THE ANISOTROPIC STRESS

We are now in position to estimate metric perturbations caused by the anisotropic stress. Consider Eq. (25) of the quasilinear approximation. As we have seen in the previous section, the initial profile of χ is a superposition of spherical waves and has a $1/r$ envelope. This form is preserved during the quasilinear stage for as long as Eq. (20) applies. However, the correlation length of χ is now determined not by the ultraviolet cutoff, but rather by the typical resonant momentum, k_* :

$$\ell = \pi/k_*. \quad (60)$$

Accordingly, instead of (57) we now have, at $r \gg \ell$,

$$\langle \chi^2 \rangle(r, t) = \langle \chi^2 \rangle_{\text{full}}(t) \ell^2(t)/r^2 . \quad (61)$$

Averaging Eq. (25) over the distribution of the random initial data and using Eq. (61), we obtain

$$r \left[\frac{(\langle \Phi \rangle - \langle \Psi \rangle)'}{r} \right]' = c \kappa^2 \frac{\langle \chi^2 \rangle_{\text{full}}}{r^2} , \quad (62)$$

where $c > 0$ is a weakly time-dependent constant of order one. Two powers of ℓ in Eq. (61) eat away two spatial derivatives in Eq. (25).

Eq. (62) is easily integrated. Using the boundary conditions $(\Phi - \Psi)(R) = 0$, $(\Phi - \Psi)'(R) = 0$, we obtain³

$$\langle \Phi \rangle - \langle \Psi \rangle = \frac{1}{2} c \kappa^2 \langle \chi^2 \rangle_{\text{full}} \left\{ \ln \frac{R}{r} + \frac{1}{2} \frac{r^2 - R^2}{R^2} \right\} . \quad (63)$$

Thus, the average gravitational potentials are more long-ranged than the average of χ^2 itself, Eq. (61).

Combining Eq. (63) with the momentum constraint (24), we can obtain estimates for $\langle \Phi \rangle$ and $\langle \Psi \rangle$ separately, at least for $r > \ell$. We will need to integrate Eq. (24) so as to remove the spatial derivative. The second term on the right hand side can only lead to a short-range, i.e. $1/r^2$, contribution, while the first term leads to a contribution that is either short-range or suppressed by several powers of M_{Pl} . So, for our present purposes we can take

$$\langle \dot{\Psi} \rangle + \mathcal{H} \langle \Phi \rangle \approx 0 . \quad (64)$$

If, as a result of parametric resonance, the field χ grows with a characteristic exponent μ , the potentials Φ and Ψ , as (63) and (64) show, grow with characteristic exponent 2μ . Then, from (64) we see that

$$\langle \Psi \rangle \sim \frac{\mathcal{H}}{\mu m_0} \langle \Phi \rangle \ll \langle \Phi \rangle , \quad (65)$$

where $m_0 \sim \sqrt{\lambda} \phi(0)$ is the frequency (in conformal time) of the inflaton's oscillations. Therefore,

$$\langle \Phi \rangle \sim \frac{\langle \chi^2 \rangle_{\text{full}}}{M_{\text{Pl}}^2} \ln \frac{R}{\ell} , \quad (66)$$

and then (65) is an estimate for $\langle \Psi \rangle$. We see that, up to a weak logarithmic factor, the average potentials are independent of R . We stress, however, that these estimates apply only in quasilinear theory.

³ Using, instead of $(\Phi - \Psi)(R) = 0$, the condition of vanishing volume average of $\Phi - \Psi$, as in Sect. III, will only change (63) by an unimportant constant.

The approximate volume independence of $\langle\Phi\rangle$ and $\langle\Psi\rangle$ during the resonant growth of χ is confirmed by numerical integrations. Integrations were done using the algorithm described in Ref. [16] and are fully nonlinear. For example, Fig. 2 shows $\delta_H\psi/\psi(R)$ for $g^2/\lambda = 110$ and $\lambda' = 0$. Here $\psi(R)$ is the conformal factor at the outer end of the grid, and $\delta_H\psi$ is defined by

$$(\delta_H\psi)^2 = 4\pi \int_{k_{\min}}^{\mathcal{H}} dk k^2 P_\psi(k) , \quad (67)$$

where $P_\psi(k)$ is the power spectrum of $\psi - \psi(R)$, and k_{\min} is the minimal lattice momentum. Fourier components of $\psi - \psi(R)$, required for calculation of the power spectrum, are determined via a sine transform. During the resonant growth of χ , the metric functions Φ and Ψ rapidly become smooth and can therefore be regarded as self-averaging. For this reason, the quantity plotted in Fig. 2 should give a good idea of the typical values of $\langle\Psi\rangle$.

Numerical values of $\langle\Phi\rangle$ and $\langle\Psi\rangle$ predicted by the estimates (66) and (65) are quite small. The usual estimate [7,8] for the maximum value of $\langle\chi^2\rangle_{\text{full}}$ is

$$\max\langle\chi^2\rangle_{\text{full}} \sim \frac{\lambda\phi^2}{g^2} \sim \frac{\lambda}{g^2} \times \frac{10^{-2}M_{\text{Pl}}^2}{\ln^2\lambda} , \quad (68)$$

so for $\lambda = 10^{-13}$ and $g^2/\lambda \sim 100$ we obtain

$$\langle\Psi\rangle \ll \langle\Phi\rangle \sim 10^{-7} \ln(R/\ell) . \quad (69)$$

From Fig. 2, we see that the lattice result by far exceeds the estimate (69), even during the stage of exponential growth, which one might want to identify with the quasilinear stage. The reason is that, because of the $1/r^2$ form of $\langle\chi^2\rangle$, the quasilinear approximation, strictly speaking, breaks down quite early on—when $\langle\chi^2\rangle$ becomes of order (68) at $r \sim \ell$. The growth of various quantities can still continue, however, because in most of the space $\langle\chi^2\rangle$ is still much smaller than that. (We discuss this point further in the next section.)

During the entire exponential growth, the two curves corresponding to different box sizes coincide with excellent precision, in agreement with the approximate volume independence of $\langle\Psi\rangle$ derived analytically. The time when the curves part corresponds to breakdown of the quasilinear theory in most of the space. At that time, volume dependence of the results is not surprising: nonlinear gravitational phenomena, such as gravitational collapse, depend on the total amount of matter available and will be more pronounced in a larger volume.

VII. FURTHER NONLINEAR EFFECTS

The driving force due to local scattering in Eq. (18) can be estimated as

$$g^2\phi\langle\chi^2\rangle \sim g^2\phi\ell^2\langle\chi^2\rangle_{\text{full}}/r^2 . \quad (70)$$

Fluctuation of ϕ caused by this driving force at $r \sim \ell$ is

$$\delta\phi \sim \frac{g^2\langle\chi^2\rangle}{\lambda\phi} . \quad (71)$$

This becomes of order ϕ when $\langle\chi^2\rangle$ reaches the value (68). After that $\langle\chi^2\rangle$ does not grow any further at $r \sim \ell$ but continues to grow at $r \gg \ell$.

From our lattice data, it appears that this later growth is a combination of the usual resonant growth and rescattering of fluctuations from smaller r , where they are already large.. As a result, the region where $\langle\chi^2\rangle$ is large propagates to larger r , so that the quasilinear form (61) no longer applies. From Eq. (25), we see that the growth of this region leads to a further growth of metric perturbations. If the total volume is large enough, these can eventually become sufficiently large to cause a gravitational instability.

The time when the resonant growth of χ has ended in most of the space corresponds to a short plateau around $t = 250$ in Fig. 2. The subsequent growth of $\delta_H\psi$ has a different origin: we interpret it is a result of an additional production of $\delta\phi$ by another, weaker resonance, due to the $\lambda\phi^4/4$ interaction. During this further growth, we observe formation of a black hole—the lapse N , as a function of r , crosses zero.⁴

The presence of the second stage of growth in Fig. 2 leads to an important conclusion. If some decay channel becomes blocked while the inflaton’s homogeneous component still has not decayed completely, the inflaton will “wait” until a new channel opens up. At least in our spherically-symmetric geometry, and in the model considered here, its decay via the new channel still causes significant metric perturbations. This conclusion holds also in the case when the decay of the inflaton into χ is blocked by a $\lambda'\chi^4/4$ self-interaction (cf. Ref. [9]): we show the result for $g^2/\lambda = 8$ (a stronger resonance) and $\lambda'/\lambda = 1000$ by short dashes in Fig. 2. In this case, the decay into χ is cut off at much smaller values of metric perturbations, and the interpretation of the second stage of growth as the $\lambda\phi^4$ resonance is unambiguous.

VIII. CONCLUSION

In this paper we have studied the influence of nonlinear effects, arising during decay of the inflaton, on metric perturbations. Although we restricted our attention to a quartic potential for the inflaton, results for other models of chaotic inflation can be obtained by similar methods, and we believe they will be similar in nature.

Early stages of the inflaton decay can be studied analytically, using the quasilinear approximation, at least under the restriction to spherical symmetry (a similar approximation may exist also in the full 3d case). The quasilinear approximation retains quadratic terms in the decay products; these are the leading terms affecting metric perturbations after inflation in the case when fields interacting with the inflaton do not have sizeable homogeneous components.

⁴ One may, and some physicists did, debate whether this is an adequate definition of a black hole. Our main argument remains the same as in Ref. [16]: for the metric (3), a sphere on which $N = 0$ cannot be crossed by a classical particle, and is therefore a horizon. On the other hand, quantum particles, or classical waves, can transport energy across the horizon (similarly to Hawking radiation); we suppose this transport is the cause of growth or evaporation of black holes in our simulations.

Using the quasilinear approximation, we have identified two effects that could potentially produce large-scale metric perturbations after the end of inflation. We have found that the first of these effects—scattering of the second field χ off the inflaton’s homogeneous component—does not cause significant super-Hubble perturbations in the quasilinear regime. However, the second type of effect—metric perturbations caused by the anisotropic stress—has been found to be important.

It is interesting to contemplate how this latter effect is able to avoid the restriction on super-Hubble physics imposed by causality. To this end, note that a spherical wave has a large degree of *angular* coherence: even for a wave with momentum much larger than the Hubble parameter, angular coherence extends over scales that are not in causal contact. Parametric amplification of such fluctuations is not restricted by causality (just as it is not restricted during inflation).

Persistence of angular coherence of fluctuations in the full three dimensions corresponds to a large mean-free path. As we have discussed in Sect. II, this is indeed likely to be the case at the early stages of reheating in weakly-coupled models. However, because restriction to spherical symmetry explicitly breaks translational invariance, it is not immediately obvious how our results can be transplanted to the full 3d case. Indeed, the r dependence of the ensemble averages $\langle\Phi\rangle$ and $\langle\Psi\rangle$ (cf. (63)) is a direct consequence of the translational non-invariance and is not expected to persist in three dimensions. Instead, the 3d picture that we have in mind is a sort of “metric turbulence”, in which a (nearly) spherically-symmetric fluctuation in a sphere of radius R_1 sits on top of a fluctuation in a sphere of a larger radius R_2 . Then, instead of $\langle\Phi\rangle$ and $\langle\Psi\rangle$ we should consider the total perturbation from all scales shorter than some R , e.g.

$$\delta_R\Phi = \langle[\Phi(\mathbf{x}) - \Phi(\mathbf{x} + \mathbf{R})]^2\rangle^{1/2}, \quad (72)$$

so for example the quasilinear result (66) will correspond to

$$\delta_R\Phi \sim \frac{\langle\chi^2\rangle_{\text{full}}}{M_{\text{Pl}}^2} \ln \frac{R}{\ell} \quad (73)$$

(with an approximately scale-invariant 3d spectrum $P_\Phi(k) \propto k^{-3} \ln k$).

Perturbation (73) is by itself quite small. We have seen, however, that in spherical symmetry a much larger perturbation is possible, due to spreading out of the region where fluctuations of χ are large. In Sect. VII, we have argued that this is indeed the mechanism of gravitational instability and formation of black holes observed in our lattice studies. Because this mechanism involves already large field fluctuations, we cannot exclude the possibility that in the full three dimensions scattering of these large fluctuations destroys angular coherence, by breaking the s -wave into higher partial waves, and the formation of black holes is prevented. However, if black holes do form in the full 3d by the mechanism identified here, the question remains what is the spectrum of metric perturbations after these black holes evaporate.

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FIGURES

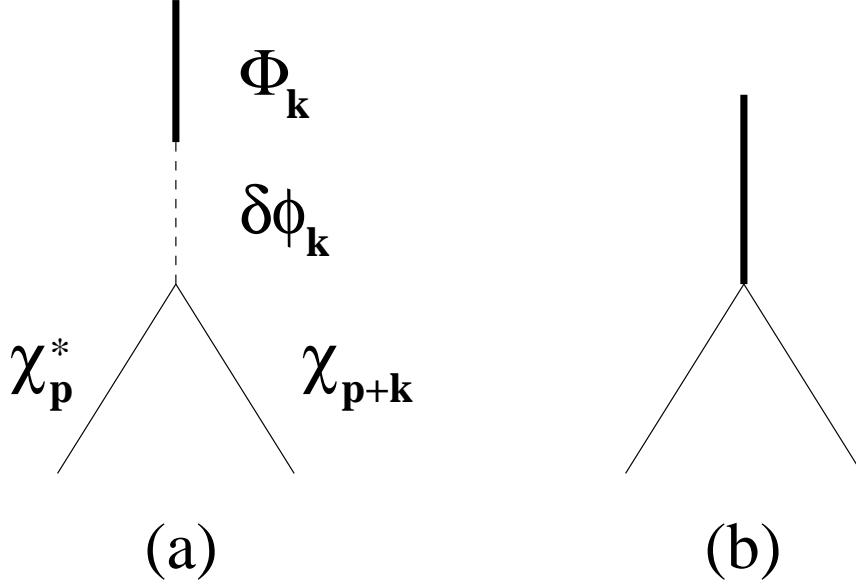


FIG. 1. Diagrams representing effects of the products χ of the inflaton decay on metric perturbations: (a) through inflaton fluctuations and (b) directly.

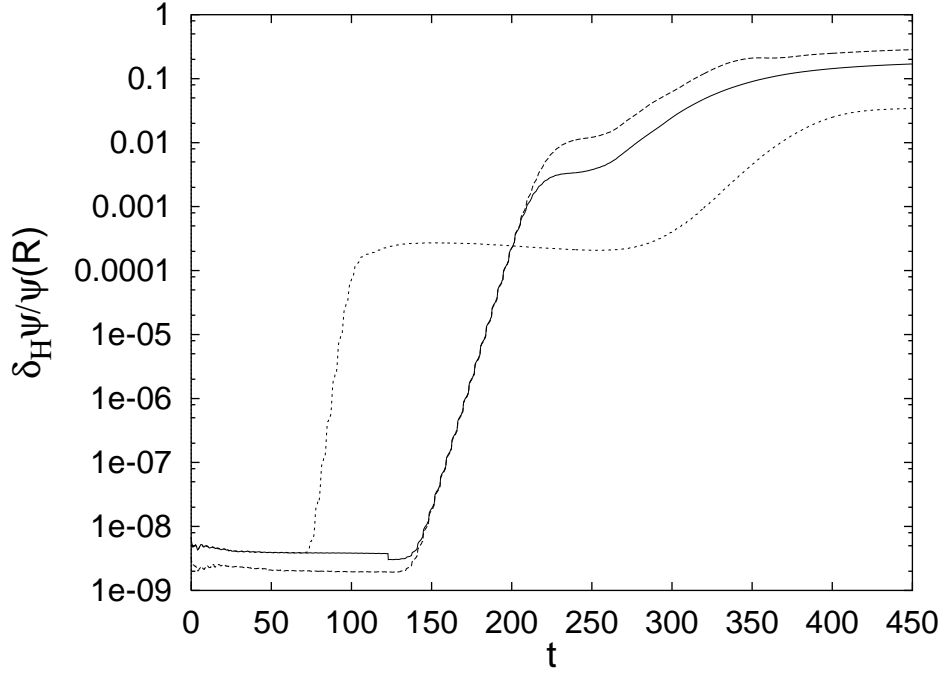


FIG. 2. Relative variation of the conformal factor over super-Hubble scales, as a function of conformal time (which is in units of $1/\sqrt{\lambda}\phi(0)$) for $g^2/\lambda = 110$, $\lambda' = 0$, $R = 500\pi$, number of grid points $n = 8192$ (solid line); $g^2/\lambda = 110$, $\lambda' = 0$, $R = 1000\pi$, $n = 16384$ (long dashes); and $g^2/\lambda = 8$, $\lambda'/\lambda = 1000$, $R = 500\pi$, $n = 8192$ (short dashes).